

Objective 1.01 b) Operations with Polynomials

A polynomial is a number, a variable, or a combination of both. A polynomial can be one or more terms.

A polynomial with one term is called a monomial. Example: $2x$

A polynomial with two terms is called a binomial. Example: $2x^2 + 4$

A polynomial with three terms is called a trinomial. Example: $3y^2 - 5y + 8$

Adding Polynomials

To add polynomials you combine like terms and write your final answers in standard form (exponents decrease from left to right).

Example 1

Simplify each sum:

a. $(2x^2 + 3 + 4x) + (-x + 4x^2 + 1)$

Combine like terms

$$2x^2 + 4x^2 = 6x^2 \quad 3 + 1 = 4 \quad 4x - x = 3x$$

Final answer

$$6x^2 + 3x + 4$$

b. $(8x - 9 - 17x^5 - 21x^2 + 14x^3) + (9 - 3x - 10x^2 + 15x^5)$

Combine like terms:

$$8x - 3x = 5x \quad -9 + 9 = 0 \quad -17x^5 + 15x^5 = -2x^5 \quad -21x^2 - 10x^2 = -31x^2$$

Final answer:

$$-2x^5 + 14x^3 - 31x^2 + 5x$$

Subtracting Polynomials

To subtract polynomials, distribute the negative (subtraction sign) then combine like terms.

Example 2

Simplify each difference:

a. $(a^3 - 4 + 18a) - (3 + 2a - 4a^2)$

Distribute the negative

$$a^3 - 4 + 18a - 3 - 2a + 4a^2$$

Combine like terms

$$-4 - 3 = -7 \quad 18a - 2a = 16a$$

Final answer

$$a^3 + 4a^2 + 16a - 7$$

b. $(-7z^3 + 3z - 1) - (-6z^2 + z + 4)$

Distribute the negative

$$-7z^3 + 3z - 1 + 6z^2 - z - 4$$

Combine like terms

$$3z - z = 2z \quad -1 - 4 = -5$$

Final answer

$$-7z^3 + 6z^2 + 2z - 5$$

You try:

Simplify.

1. $(d^3 + 2) - (15d + 2d - 9)$

2. $(-4v^5 + 7v^4 + v - v^2) + (2v - v^2 - 4v^5)$

3. $(y^2 - 4y + 3) - (-y^2 - 3)$

4. $(25x^5 - 9 + 14x) + (-2x - 10x^5 + 5)$

Answers: 1) $d^3 - 17d + 11$ 2) $-8v^5 + 7v^4 - 2v^2 + 3v$ 3) $2y^2 - 4y + 6$ 4) $15x^5 + 12x - 4$

Distributing Monomials

Examples:

Simplify

a. $2x(x^3 + 4x + 2)$

Distribute $2x$ to each term

$$2x(x^3) = 2x^4 \quad 2x(4x) = 8x^2 \quad 2x(2) = 4x$$

Final answer

$$2x^4 + 8x^2 + 4x$$

b. $-9f^4(5f - 3 + 2f^2 - 9f^3)$

Distribute $-9f^4$ to each term

$$-9f^4(5f) = -45f^5 \quad -9f^4(-3) = 27f^4 \quad -9f^4(2f^2) = -18f^6 \quad -9f^4(-9f^3) = 81f^7$$

Final answer

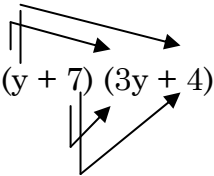
$$81f^7 - 18f^6 - 45f^5 + 27f^4$$

Multiplying Binomials using FOIL

When you are multiplying binomials you can use the FOIL method. F-first O-outer I-inner L-last This term is a memory device for applying the Distributive Property to the product of two binomials.

Example:
Simplify

a. $(y + 7)(3y + 4)$



F-first terms: $y(3y) = 3y^2$

O-outer terms: $y(4) = 4y$

I- inner terms: $7(3y) = 21y$

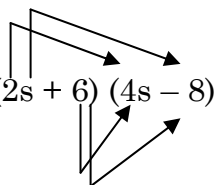
L-last terms: $7(4) = 28$

Combine OI = $25y$

Final answer

$$3y^2 + 25y + 28$$

b. $(2s + 6)(4s - 8)$



F: $8s^2$

O: $-16s$

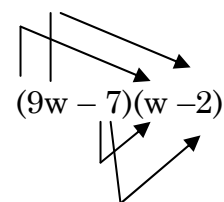
I: $24s$

L: -48

Final answer:

$$8s^2 + 8s - 48$$

c. $(9w - 7)(w - 2)$



$$9w^2 - 18w - 7w + 14$$

Final answer: $9w^2 - 25w + 14$

You try:

1. $-x^4(17x^2 - 3x^3 + 4x - 2)$

2. $3x^2(3 + 2x - 15x^2 + \frac{1}{3}x^3)$

3. $(a - 4)(6a - 9)$

4. $(-y - 12)(5 + y)$

Answers: 1) $3x^7 - 17x^6 - 4x^5 + 2x$ 2) $x^5 - 45x^4 + 6x^3 + 9x^2$ 3) $6a^2 - 33a + 36$
4) $-y^2 - 17y - 60$