Lesson: Solving one-variable equations using inverses.

Eighth Grade Objective: **5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.**

Lesson:

In real life, we are frequently placed in situations where we need to find a piece of information we are not given directly. Many times, an algebraic equation can be written to solve for the missing piece of information.

Write an equation to represent the following situations:

1. Write an equation to determine the original cost of a meal if the total bill, including a 15% tip, is $6.90.

We can represent the original cost of the meal with a variable, m.
The total is computed by adding the cost of the meal plus 15% of the meal, or
$6.90 = m + 0.15m$
-or-
$6.90 = 1.15m$

2. Jen makes $250 this month at work. She gets $50 plus $40 for each new account she opens. Write and equation to solve for how many new accounts she has opened this month.

We can represent the new accounts with a variable, a.
$250 = 50 + 40a$

3. The perimeter of a rectangle is 30 cm. The length is 12 cm. Write an equation to determine the width.

We can represent the length with the variable, l.
Recall, perimeter of a rectangle can be determined using the formula: $p = 2l + 2w$
$30 = 2(12) + 2(w)$
$30 = 24 + 2w$

To solve single-variable algebraic equations, we use inverse operations and we go in reverse order of operations.

Some common inverse operations:

Operation	**Inverse**
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Square	Square Root
Square Root	Square

When there is more than one operation in a problem, we "undo" the operations in reverse order of operations. It is helpful to think "How would I solve this problem if I knew what the variable was?" and then think backwards.

Keep in mind that what you do to one side, you MUST do to the other to keep the equations balanced.

Let's solve for each of the missing variables in examples 1-3 above:

1. $6.90 = 1.15m$

Think: If I knew what m was, I would multiply it by 1.15.

To undo multiplication, I divide both sides by 1.15:

$$\frac{6.90}{1.15} = \frac{1.15m}{1.15}$$

$$6 = m$$

The cost of the meal, before tip, was $6.

2. $250 = 50 + 40a$

Think: If I knew what a was, I would multiply by 40, then add 50.

Now, do the inverse operations in reverse order:

$$250 = 50 + 40a$$

Subtract 50:

$$-50 \quad -50$$

$$\frac{200}{40} = \frac{40a}{40}$$

Divide by 40:

$$a = 5$$

She opened 5 new accounts this month.

3. $30 = 24 + 2w$

Think: If I knew what w was, I would multiply by 2, then add 24.

Now, do the inverse operations in reverse order:

$$30 = 24 + 2w$$

Subtract 24:

$$-24 \quad -24$$

$$6 = 2w$$

Divide by 2: $3 = w$

The width of the rectangle is 3.

Solving equations with multiple steps:

We follow the same steps as above, there are just more of them!

1. $2(4d - 7) = 18$

First, let's distribute the 2: $8d - 14 = 18$

Think: If I knew what d was, I would multiply by 8 and then subtract 14.

Now, do the inverse operations in reverse order:

$8d - 14 = 18$

Add 14: $+14 \quad +14$

$\frac{8d}{8} = \frac{32}{8}$

Divide by 8:

$\boldsymbol{d = 4}$

2. $5(\frac{1}{2}f + 3) = 25$

First, let's distribute the 5: $(5/2)f + 15 = 25$

Think, If I knew what f was, I would subtract 15 and divide by (5/2).

Now, do the inverse operations in reverse order:

$(5/2)f + 15 = 25$

Subtract 15: $-15 \quad -15$

$\frac{(5/2)f}{(5/2)} = \frac{10}{(5/2)}$

Divide by (5/2):

$\boldsymbol{f = 4}$

3. $-2 + (3x)/4 = 1$

Think: If I knew what x was, I would multiply by 3, divide by 4 and add -2.

Now, do the inverse operations in reverse order:

$-2 + (3x)/4 = 1$

Subtract -2: $-(-2) \quad -(-2)$

$(3x)/4 = 3$

Multiply by 4: $4 * (3x)/4 = 4 * 3$

$\frac{(3x)}{3} = \frac{12}{3}$

Divide by 3:

$\boldsymbol{x = 4}$

When there are variables on both sides, it is important first to get all like terms together. That means we will distribute anything there is to distribute, then we will combine any like terms within the sides, then we will use inverse operations to combine all the terms, across sides of the equals sign, with variables in them. Then we will solve the equation like we have in the previous examples.

1. $8x - 1 = 23 - 4x$

There is nothing to distribute.

There is nothing that can be combined on either side of the equals sign (you can't add $8x$ and 1 since they are not like terms. You also cannot subtract 23 and $4x$ because they are not like terms.).

So we either need to move the 8x by subtraction or we need to move the $-4x$ by addition. It does not matter which you choose to do, but it is easier to add $4x$, so that is what we will do.

$$8x - 1 = 23 - 4x$$

Add 4x to both sides: $+4x \qquad + 4x$

$$12x - 1 = 23$$

Think: If I knew what x was, what would I do? I would multiply by 12, then subtract 1. So, now do the inverse operations in the reverse order:

$$12x - 1 = 23$$

Add 1: $+ 1 \quad +1$

$$\frac{12x}{12} = \frac{24}{12}$$

Divide by 12:

$$\boldsymbol{x = 2}$$

2. $2m - 5(m + 1) = 3m + 1$

First, let's distribute:

$$2m - 5(m + 1) = 3m + 1$$

$$2m - 5m - 5 = 3m + 1$$

On the left side, we can combine like terms:

$$-3m - 5 = 3m + 1$$

Now, we either need to add $3m$ to both sides or subtract $3m$ from both sides. Let's add $3m$ since it is easier:

$$-3m - 5 = 3m + 1$$

$+3m \qquad +3m$

$$-5 = 6m + 1$$

Think: If I knew what m was, I would multiply by 6 and add 1

So, now do the inverse operations in reverse order:

$$-5 = 6m + 1$$

Subtract 1: $- 1 \qquad -1$

$$\frac{-6}{6} = \frac{6m}{6}$$

Divide by 6:

$$\boldsymbol{m = -1}$$

Try these on your own!

1. The volume of a rectangular prism is 600 cubic inches. The length and width are both 20. Write and solve an equation to determine the height of the prism.

2. James makes $200 less than four times Mike's salary. If James makes $1400 a month, write and solve an equation for how much Mike makes.

3. Solve for y: $2(-3y + 10) = 32$

4. Solve for c: $9c - 12 = 8c - 3$

5. Solve for w: $4w + 6 + 3w = 2w + 31$

Check your answers:

1. We can represent the height using the variable, h.
Recall that $V = lwh$, so:
$600 = 20 * 20 * h$
$600 = 400h$
Now, think: If I knew what h was, I would multiply by 400.
So, now I do use the opposite operation:
Divide by 400:

$$\frac{600}{400} = \frac{400h}{400}$$
$$\boldsymbol{h = 1.5}$$

2. We can represent Mike's salary with the variable, m.
James makes $4m - 200$. Knowing that James' salary is $1400, we know $1400 = 4m - 200$
Think: If I knew what m was, I would multiply by 4 and subtract 200.
Now, use the inverse operations in reverse order:
Add 200:

$$1400 = 4m - 200$$
$$+200 \qquad +200$$
$$\frac{1600}{4} = \frac{4m}{4}$$
$$200 = m$$

Divide by 4:

Mike's salary is $200/month.

3. First we distribute:

$$2(-3y + 10) = 32$$
$$-6y + 20 = 32$$

Think: if I knew what y was, I would multiply by -6 and add 20.
Now, use the inverse operations in reverse order:
Subtract 20:

$$-6y + 20 = 32$$
$$-20 \quad -20$$
$$\frac{-6y}{-6} = \frac{12}{-6}$$
$$\boldsymbol{y = -2}$$

Divide by -6:

4. We have nothing to distribute and we have no like terms to combine, so we can either subtract 9*c* from both sides, or subtract 8*c* from both sides. Subtracting 8*c* is easier, so we will start there.

$$9c - 12 = 8c - 3$$
$$-8c \qquad -8c$$
$$c - 12 = -3$$

Now, use the inverse operation: Add 12:

$$+12 \quad +12$$
$$\boldsymbol{c = 9}$$

5. We have nothing to distribute, but we can combine like terms:

$$4w + 6 + 3w = 2w + 31$$
$$7w + 6 = 2w + 31$$

Now we can either subtract 7*w* from both sides or subtract 2*w* from each side. Subtracting 2*w* is easier, so we'll start there.

$$7w + 6 = 2w + 31$$
$$-2w \qquad -2w$$
$$5w + 6 = 31$$

Think: If I knew what *w* was, I would multiply by 5 and subtract 6.

Now, use the inverse operations in reverse order: $5w + 6 = 31$

Subtract 6: $\quad -6 \quad -6$

$$\frac{5w}{5} = \frac{25}{5}$$

Divide by 5:

$$\boldsymbol{w = 5}$$

Quiz yourself!

1. A Labrador Retriever eats 3 pounds more than twice the amount a Poodle eats. A Labrador Retriever eats 8 pounds of food per day. Write and solve an equation for how much a Poodle eats.

2. Ron makes $100 more than half of Sarah's weekly salary. Ron makes $300 per week. Write and solve an equation for the amount of Sarah's weekly salary.

3. Solve for *x*: $(13x - 3) / 4 = 9$

4. Solve for *n*: $-2(4n - 3) = -4n + 10$

5. Solve for *x*: $6x + -2 + 2x = -2 + 4x + 8$

Check your answers:

1. $8 = 3 + 2p$; $p = 2.5$ pounds

2. $300 = 100 + \frac{1}{2}s$; $s = 400$

3. $x = 3$

4. $n = -1$

5. $x = 2$